Quantitative Finance

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Other Annuities Certain

ISEG 100 1911-2011 ANOS

100 ANOS A PENSAR NO FUTURO

Deferred annuities

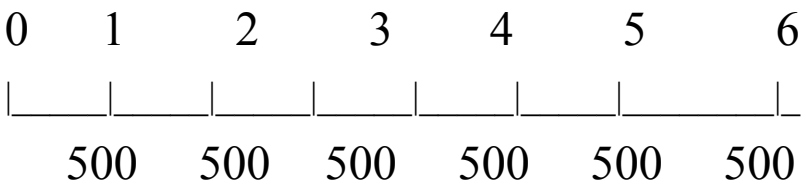
An Annuity with the present value located two or more periods before the first payment is called a deferred annuity.

$PV = R \times a_{n|i} (1+i)^{-m}$; where **m** is the number of intervals of deferment, in this example m=1

$FV = PV (1+i)^{n}$

Comparing ordinary annuities and deferred annuities

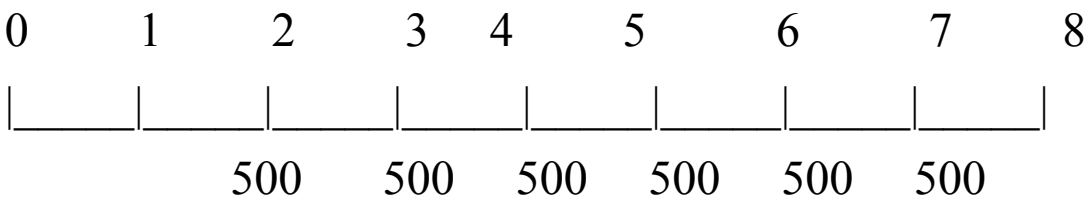
Ordinary annuity (i=0.01)



PV0= 500 x a6|0.01 = 2897.74

FV**6**= 500 x S6|0.01 = 3076.01

Deferred annuity ((i=0.01; m=1



PV0= 500 x a6|0.01 x (1+0.01)-1 = 2869,05

FV**8**= 500 x S6|0.01 = 3076.01

Forborne annuities

An Annuity with the future value located p periods after the last payment.

R R R R R R FV

0 1 2 3 4 5 6.... p

$FV = R \times S_{n|i} (1+i)^p$; where p is the number of intervals between the last payment and FV.

PERPETUITIES

A perpetuity is an annuity with infinite term in which the periodic payments are the interest earned during the previousinterest period. A perpetuity has only a present value.

Ordinary perpetuity

$$a_{\infty|i} = \lim_{n \to \infty} (a_{n|i}) = 1/i$$

$$PV = R/i$$

Perpetuity due

$$PV = R/i \times (1+i) = R + R/i$$

Deferred perpetuity (m periods)

$$PV = R/i \times (1+i)^{-m}$$

PRESENT VALUE – GROWING ARITHMETIC PROGRESSION

Arithmetic progression: This is a sequence which has a starting number C and successive numbers are obtained by adding a number h (called the common difference or constant):

$$c,\ c + h,\ c + 2h,\ \cdots,\ c + (n-1)h$$

Ex: 1000, 1500, 2000, 2500 - First term is 1000 and the constant is 500.

Present value when first term is C and the constant is h:

$$PV = (C-h)\ a_{n|i} + h\ (Ia)_{n|i}$$

Where $(Ia)_{n|i} = [\ddot{a}_{n|i} - n\ (1+i)^{-n}] / i$

PRESENT VALUE – DECREASING ARITHMETIC PROGRESSION

Ex: 2500, 2400, 2300, 2200 - Last term is 2200 and the constant is 100.

Present value when last term is D and the constant is h:

$$PV = (D-h)\, a_{n|i} + h\, (Da)_{n|i}$$

Where $(Da)_{n|i} = (n - a_{n|i}) / i$

PRESENT VALUE – GEOMETRIC PROGRESSION

Geometric progression: The geometric progression has a starting number C and successive terms are obtained by multiplying by a common ratio r called the factor:

$$C,\ Cr,\ Cr^2,\ \cdots,\ Cr^{(n-1)}$$

Ex: 100, 200, 400, 800 - First term is 100 and the factor is 2

Factor > 1 growing progression.

Factor < 1 decreasing progression

Present value when first term is C and the factor is r:

$$PV = C\ [\ 1\text{-}rn\ (1+i)\text{-}n] / (1+i\text{-}r)$$